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1.1.1 Properties of true errors

True errors can be found when we know the true value of the measured value. In geodetic measurements and in any other measurements, the true value of the measured quantity is unknown, therefore the most probable value of this quantity is taken, the sum of probable errors must be zero, the sum of true errors must be zero, therefore it is necessary to investigate the probable and true errors in an algebraic circle .

The nature of true errors is manifested in closed systems, each (any) element of a closed system is neutral and opposite to all other elements and can be the beginning and end of the system. Therefore, a simple, closed system can be considered as an algebraic circle of elements, the dimensions of which are not measured precisely, but with some error.

Definition 1. The set of all elements of a closed system is called a circle ***K*** if the algebraic sum of the values of these elements is a true physical quantity.

Definition 2. The set of all elements of a closed system is called a circle ***M*** if the algebraic sum of the values of these elements is zero.

In circle ***K*** and circle ***M***, we consider their own subsets, arcs D and $\overline{D}$ which are opposite to each other and such that

$$D \subset K, \overline{D} = K \setminus D ; \; D \subset M, \overline{D} = M \setminus D. \quad (1)$$

Theorem. If the elements of the system create circle ***K*** or circle ***M***, then the algebraic sum of their true errors is zero, and the algebraic sums of errors of the elements of any two opposite arcs D and $\overline{D}$, circle ***K*** or circle ***M***, will be the same in magnitude and have opposite signs.

Proof. Let $X_1, X_2, \ldots, X_n$ are the true values of the elements of circle ***K*** or circle ***M***; $x_1, x_2, \ldots, x_n$ – approximate (measured) values of the elements; $\Delta_1, \Delta_2, \ldots, \Delta_n$ are the true errors of the elements.

If C is the algebraic sum of the true values of the elements of the circle ***K***, or $C = 0$ in the circle ***M***, then we can write

$$\begin{cases} \Delta_1 = x_1 - X_1 \\ \Delta_2 = x_2 - X_2 \\ \Delta_3 = x_3 - X_3 \\ \quad \dots . \\ \Delta_n = x_n - X_n \end{cases} \qquad (2)$$

Let's add the right and left sides of these equations, we get:

$$\Delta_1 + \Delta_2 + \ldots + \Delta_n = x_1 + x_2 \ldots + x_n - (X_1 + X_2 + \ldots + X_n).$$

Here the algebraic sum of the elements will be

$$\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} X_i = C, \qquad (3)$$

then the sum of the true errors of the elements will be

$$\sum_{i=1}^{n} \Delta_i = 0. \qquad (4)$$

Let's divide circle ***K*** or circle ***M*** into two opposite arcs D and $\overline{D}$, introduce the notation

$$\begin{cases} X_1 + X_2 + \cdots + X_k = X_D \\ X_{k+1} + X_{k+2} + \cdots + X_n = X_{\overline{D}} \\ x_1 + x_2 + \cdots + x_n = x_D \\ x_{k+1} + x_{k+2} + \cdots + x_n = x_{\overline{D}} \\ \Delta_1 + \Delta_2 + \cdots + \Delta_k = \Delta_D \\ \Delta_{k+1} + \Delta_{k+2} + \cdots + \Delta_n = \Delta_{\overline{D}} \end{cases} . \qquad (5)$$

Then the true errors of arcs D and $\overline{D}$ will be determined

$$\Delta_D = x_D - X_D \qquad (6)$$

$$\Delta_{\overline{D}} = x_{\overline{D}} - X_{\overline{D}} \qquad (7)$$

Let's add the left and right parts of equations (6) and (7), we get

$$\Delta_D + \Delta_{\overline{D}} = x_D + x_{\overline{D}} - (X_D + X_{\overline{D}}).$$

But by definition

$$x_D + x_{\overline{D}} = X_D + X_{\overline{D}},$$

ago

$$\Delta_D = -\Delta_{\overline{D}},$$

$$|\Delta_D| = |-\Delta_{\overline{D}}|,$$

or

$$\Delta_1 + \Delta_2 + \cdots + \Delta_k = -(\Delta_{k+1} + \Delta_{k+2} + \cdots + \Delta_n),$$

$$|\Delta_1 + \Delta_2 + \cdots + \Delta_k| = |-(\Delta_{k+1} + \Delta_{k+2} + \cdots + \Delta_n)|$$

Let's take all equal internal (or external) angles $\beta_1, \beta_2, \ldots, \beta_n$ of a closed polygonometric (or theodolite) course, which create a circle ***K***, because their true sum is known: $C = \pi(n-2)$, or $C = \pi(n+2)$.

If $\Delta_1, \Delta_2, \ldots, \Delta_n$ are the true errors of the angles, then by the theorem we get

$$\sum_{i=1}^{n} \Delta_i = 0.$$

For example, let's take all equalized excesses $h_1, h_2, \ldots, h_n$ of a closed leveling stroke. The set of all excesses creates a circle ***M***, because the algebraic sum of excesses must be equal to zero, that is, $\sum_{i=1}^{n} h_i = 0$. Suppose that $\Delta_1, \Delta_2, \ldots, \Delta_n$ are true errors of excesses, then also

$$\sum_{i=1}^{n} \Delta_i = 0.$$

When the circle ***M*** is divided into two opposite arcs *D* and $\sum_{i=1}^{n} \Delta_i = 0$, first we will take any excesses from ***M***, in the number of one to *n* -1 excesses, regardless of the order of excesses in the course. This set of excesses will create the arc *D*, and the set of excesses that will remain will create the opposite arc $\sum_{i=1}^{n} \Delta_i = 0$, from the theorem it follows that the algebraic sum of all true errors of the excesses of the closed stroke is equal to zero $\sum_{i=1}^{n} h_i = 0$, and the algebraic sums of true errors of arcs *D* and $\overline{D}$ have equal moduli and opposite signs.

To determine the planned coordinates x and y of points of polygonometric (theodolite) moves, coordinate increments are determined by formulas

$$\begin{cases} \Delta_x = d \cdot \cos\alpha \\ \Delta_y = d \cdot \sin\alpha \end{cases} \tag{8}$$

where *d* is the measured distance (horizontal laying) between adjacent points, the coordinates of which are determined; α are equalized directional angles, which are reduced to circle ***M***.

We equalize the increments of the coordinates, bringing them to the circle ***M***, in such a way that

$$\begin{cases} \sum_{i=1}^{n-1} \Delta x_i = 0 \\ \sum_{i=1}^{n-1} \Delta y_i = 0 \end{cases} \tag{9}$$

where n is the number of defined points.

By making corrections to the coordinate increments, we thereby equalize the lengths of the measured distances between adjacent points.

Note that the circles ***M*** in the same way form the equalized increments of the coordinates of the points of a closed (and not necessarily closed, if it can be reduced to a circle M, that is, all geodesic constructions are closed among themselves) polygonometric (theodolite) course, the sum of which is zero.

True errors do not accumulate in the sums of elements of a closed equalized system. If the angles of a closed polygonometric (theodolite) course are precisely measured, and then they are equalized, and if the mean square error of the alignment of the angles m_β is known, then all the directional angles of the sides of the polygonometric (theodolite) course have the same mean square error $m_\alpha = m_\beta$. Similarly, if in a closed leveling course the excesses with the mean square error m_h of the leveled excess are exactly measured, then all the heights (marks) of the points (benchmarks) will have the same mean square error $m_H = m_h$. If the distances between the points are exactly measured and the mean square error of determining the distances m_d is obtained during alignment, then the mean square errors of the coordinate increments with equal influence of the accuracy of linear and angular measurements will be

$$\begin{cases} m_{\Delta\text{x}} = \sqrt{\frac{m_d^2}{2} + \frac{d^2 \cdot m_\alpha^2}{2 \cdot \rho^2}}, \\ m_{\Delta y} = \sqrt{\frac{m_d^2}{2} + \frac{d^2 \cdot m_\alpha^2}{2 \cdot \rho^2}} \end{cases} \qquad (10)$$

These formulas include mean squared errors in determining the lengths of lines and angles

The given theorem establishes a feature of the distribution of true errors. It is proved that the algebraic sum of the true errors of the elements of the closed equalized system is zero

1.1.2 Average variance as a general characteristic of the dispersion of a random variable

A new method of measuring accuracy assessment is under consideration. It differs from traditional methods in that, as a result of processing measurements of one quantity, not only the mean squared error is obtained, but also the mean squared errors of all measurements.

Properties of average dispersion

A new approach to assessing the accuracy of measurements was initiated in the work [1], in which the dispersion of the value of a discrete random variable is given. It is proved that the average variance and general variance have the following dependence:

$$\nu_0^2 = 2\sigma^2 \tag{11}$$

An unsolved problem is establishing the properties of the average variance.

The goal is to establish the essence of this dispersion characteristic of the discrete quantity *X*.

We will consider a value that has value as one that is scattered not only relative to the general average value, but also relative to individual values of this random variable. If the center of dispersion of a random variable is taken as a mathematical expectation, then the characteristic of its dispersion will be the general dispersion

$$\sigma^2 = E[(X-\mu)^2] = \sum_x (x-\mu)^2 f(x), \tag{12}$$

where *f* (x) is the probability distribution function [5].

If we take some value x_i from the group *G* of the general population [3] as the center of dispersion of the value *X*, then we arrive at the following definition of the dispersion of the value x_i of a random variable:

$$\sigma_{xi}^2 = E[(X-x_i)^2] = \sum_x (x-x_i)^2 f(x). \tag{13}$$

A comparison of dependencies (12), (13) shows that general variance and variance have the same algebraic representation. However, in reality, these two characteristics have different properties. To prove this, consider the aggregates in which these two characteristics are defined.

The variance σ^2 is determined by the aggregate (volume *k* +1):

$$C' = (\mu, x_1, x_2, \dots, x_i, \dots, x_{k-1}, x_k),$$

The dispersion σ_{xi}^2 is calculated by the general population

$$C \quad = (x_1, x_2, \dots, x_i, \dots, x_{k-1}, x_k).$$

General dispersion

$$\sigma^2 = \frac{1}{k}\sum_{i=1}^{k}(x_i-\mu)^2$$

is the square of the mean squared difference of the general average μ and the indicators of all dimensions, since in this equation the value $\sum_x (x_j - \mu)^2$ is the sum k of the squares of the differences, and the variance

$$\sigma_{xi}^2 = \frac{1}{k} \sum_{xj} (x_j - x_i)^2 \quad (j \mid i = 1, 2, \ldots, k,\ j \neq i\ ) \tag{14}$$

is not the square of the mean squared difference of the value x_i of the value X and the values complementary x_i to X, because $\boldsymbol{\sum_{xj} (x_j - x_i)^2}$ in dependence (14) is formed k - 1by the squares of the differences.

It happens that the general population has a large volume k_G of the group G. Then the variance of the value of the quantity is calculated according to the theorem [1]

$$\sigma_x^2 = \sigma^2 + (x - \mu)^2 \tag{15}$$

The general variance is determined by the following theorem [1]:

$$\sigma^2 = \frac{\sum_{i=1}^{k_G} \sigma_{xi}^2 f(x_i)}{2} \quad (i = 1, 2, \ldots, k_G), \tag{16}$$

where $f(x_i)$ is the probability of the value x_i of the group G. Since the volume k_G is equal to the volume of "values" of the quantity X, it follows from dependence (16):

$$\sigma^2 = \frac{\sum_x \sigma_x^2 f(x)}{2}.$$

Taking into account dependence (11), we obtain the following rule for determining the average variance of a quantity X:

$$\nu_0^2 = \sum_x \sigma_x^2 f(x).$$

Assertion 1. The average variance ν_0^2 is the dispersion field σ_x^2 of a value X relative to the following values: $x = \mu - \sigma$, $x = \mu + \sigma$.

Proof. If there is a general set of measurements of one quantity, then, according to the axiom of the theory of measurement errors [3], a random quantity X acquires a set of values in such a limited closed interval:

$$\left[x_{min} - \frac{[Q]}{2}, x_{max} + \frac{[Q]}{2}\right] \tag{17}$$

where x_{min}, x_{max} is the smallest and largest value of measurements; $[Q]$ – degree of quantization of measurements [4].

Therefore, the value X can have the following values within: $x = \mu - \sigma$, $x = \mu + \sigma$. We determine the dispersion σ_x^2 of these values according to rule (5), and obtain

$$\sigma_x^2 = \sigma^2 + (x-\mu)^2 = \sigma^2 + (\mu - \sigma - \mu)^2 = 2\sigma^2 = \nu_0^2;$$

$$\sigma_x^2 = \sigma^2 + (x-\mu)^2 = \sigma^2 + (\mu + \sigma - \mu)^2 = 2\sigma^2 = \nu_0^2,$$

then the statement is proved.

The average variance ν_0^2 is determined in the general aggregates of measurements, i.e. in aggregates characterized by fullness *F*=1 and such a quality factor *Q* that allows reliably establishing the probability distribution function *f* (*x*) [3]. If, in the process of measuring one quantity, a set *k* of measurements is collected, which has a large volume and a significant quality factor *Q*, but the fullness of the set is *F*<1, that is, the projection of the set is not a complete set *G* of measurements, then this set of measurements can be considered only as a random sample representing the general set. Then the sample mean $\bar{x}$ (simple arithmetic mean) and the approximate value $\nu^2$of the dispersion characteristicν_0^2 of the quantity *X* [1] are determined according to formula (11)

$$\nu^2 = 2s^2, \qquad (18)$$

were s^2 – це вибіркова дисперсія [2]. Дисперсія ν^2 є оцінкою для середньої дисперсії ν_0^2. Від відповідності (15) приходимо до такого правила [1]:

$$s_x^2 = s^2 + (x - \bar{x})^2. \qquad (19)$$

Since dependencies (15), (19) are the same algebraic correspondences, the value $\boldsymbol{\nu^2}$ is equal to the sampling variance $\boldsymbol{s_x^2}$ of the values $\boldsymbol{x = \bar{x} - s, x = \bar{x} + s}$ of the sample .

Example 1. The table shows a series of distribution of the general population of measurements of excess by growth between two benchmarks of the leveling course [1].

It is necessary to determine the variances $\boldsymbol{\sigma_s^2, \sigma_t^2}$ of the values $\boldsymbol{x_s = \mu + \sigma, x_t = \mu - \sigma}$ of the random variable and compare these variances with the average variance $\boldsymbol{\nu_0^2}$.

Table The series of distribution of the general set of measurements

X h(мм)	$x_{(1)}$ 1,3	$x_{(2)}$ 1,4	$x_{(3)}$ 1,5	$x_{(4)}$ 1,6	$x_{(5)}$ 1,7	$x_{(6)}$ 1,8	$x_{(7)}$ 1,9
$f(x)$ $p(x)$	$f(x_{(1)})$ 0,01	$f(x_{(2)})$ 0,03	$f(x_{(3)})$ 0,18	$f(x_{(4)})$ 0,49	$f(x_{(5)})$ 0,23	$f(x_{(6)})$ 0,04	$f(x_{(7)})$ 0,02
k_G	1	2	3	4	5	6	7

The general population has the volume of measurements k = 100, and the volume of «values» of the general population k_G = 7. The degree of quantization of measurements [Q] = 0.1 mm. The range of measurements R = 1.9 – 1.3 = 0.6 mm. Characteristics of the position of the random variable μ=1.60 mm. Characteristics of its dispersion: $\sigma^2 = 0{,}0095$ мм^2 ; $\nu_0^2 = 2\sigma^2 = (2)(0{,}0095) = 0{,}019$ мм^2.

A random variable X acquires a set of values in such a limited closed interval:

$$x_{min} - \frac{[Q]}{2};\ x_{max} + \frac{[Q]}{2};\ x_1 - \frac{[Q]}{2};\ x_7 + \frac{[Q]}{2} = [1.25 \div 1.95],$$

accordingly, the confidence interval for the quantity X has the form with a confidence probability of one:

$$P(1{,}25 \text{ мм} \le X \le 1{,}95 \text{ мм}) = 1.$$

We will get:

$$x_s = \mu + \sigma = 1.61 + 0{,}0975 = 1{,}7075 \text{ мм};\ x_t = \mu - \sigma = 1{,}61 - 0{,}0975 = 1{,}5125 \text{ мм};$$

$$\sigma_{x_s}^2 = \sigma^2 + (x_s - \mu)^2 = 0{,}0095 + (1{,}7075 - 1{,}61)^2 = 0{,}019 \text{ мм}^2 = \nu_0^2.$$

$$\sigma_{x_t}^2 = \sigma^2 + (x_t - \mu)^2 = 0{,}0095 + (1{,}5125 - 1{,}61)^2 = 0{,}019 \text{ мм}^2 = \nu_0^2.$$

Consider the density function of the distribution of a quantity X with mean value μ and variance $\boldsymbol{\sigma^2}$ for the normal distribution law [5]

$$n(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(1/2)[(x-\mu)/\sigma]^2}, \qquad -\infty < x < \infty. \tag{20}$$

Statement 2. If the value X is normally distributed, has an average variance $\boldsymbol{\nu_0^2}$, then for values $\boldsymbol{x = \mu \pm \sigma}$ the density function of the distribution of a random value will have the following form for the values: $n(x = \mu \pm a\sigma; \mu, \nu_0) = \frac{1}{\sqrt{\pi}\nu_0} e^{-a^2/2}.$

Proof. Theorem (15) implies that

$$(x-\mu)^2 = \sigma_x^2 - \sigma^2. \tag{21}$$

Taking into account dependencies (11), (21), we obtain

$$-(1/2)[(x-\mu)/\sigma]^2 = -(1/2)(\sigma_x^2 - \sigma^2)/\sigma^2 = -\sigma_x^2/2\sigma^2 + \sigma^2/2\sigma^2 =$$

$$1/2 - \sigma_x^2/v_0^2.$$

Therefore, the function (20) is reduced to the following form:

$$n(x;\mu,v_0) = \frac{1}{\sqrt{\pi}v_0} e^{1/2-\sigma_x^2/v_0^2}, \quad -\infty < \boldsymbol{x} < \infty \tag{22}$$

For the values $\boldsymbol{x = \mu}$, $\boldsymbol{x = \mu \pm \sigma}$, $\boldsymbol{x = \mu \pm 2\sigma}, \cdots, \boldsymbol{x = \mu \pm a\sigma}$ of the random variable, in accordance with the rules (11), (15), we obtain the following variances:

$$\sigma_{x=\mu}^2 = 1/2v_0^2;\ \sigma_{x=\mu\pm\sigma}^2 = v_0^2;\ \sigma_{x=\mu\pm2\sigma}^2 = 5\sigma^2 = (5/2)v_0^2;\ldots$$

$$\ldots\sigma_{x=\mu\pm a\sigma}^2 = (a^2+1)\sigma^2 = [(a^2+1)/2]v_0^2.$$

For the variances of the given values, we find the densities of the normal distribution of the random variable by rule (22).

$$\left.\begin{array}{l}
n(x=\mu;v_0) = \dfrac{1}{\sqrt{\pi}v_0} e^{1/2-(1/2)v_0^2/v_0^2} = \dfrac{1}{\sqrt{\pi}v_0};\\
n(x=\mu\pm\sigma;v_0) = \dfrac{1}{\sqrt{\pi}v_0} e^{1/2-v_0^2/v_0^2} = \dfrac{1}{\sqrt{\pi}v_0} e^{-1/2};\\
n(x=\mu\pm2\sigma;v_0) = \dfrac{1}{\sqrt{\pi}v_0} e^{1/2-(5/2)v_0^2/v_0^2} = \dfrac{1}{\sqrt{\pi}v_0} \mathrm{e}^{-2};\\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
n(x=\mu\pm a\sigma;v_0) = \dfrac{1}{\sqrt{\pi}v_0} e^{1/2-[(a^2+1)/2]v_0^2/v_0^2} = \dfrac{1}{\sqrt{\pi}v_0} e^{-a^2/2}.
\end{array}\right\} \tag{23}$$

The statement is proved.

Example 2. In example 1, the general population has the following characteristics: μ = 1.60 *mm*; σ = 0.0975 *mm*; $\boldsymbol{v_0^2 = 0.019\ mm^2}$. It is necessary to calculate the density ratio of the distribution of a random variable for its values $\boldsymbol{x = \mu}$, $\boldsymbol{x = \mu \pm \sigma}$, $\boldsymbol{x = \mu \pm 2\sigma}, \cdots, \boldsymbol{x = \mu \pm a\sigma}$ using formulas (23): .

The following values of the distribution density of the random variable were calculated:

$$n(x=\mu;v_0)=\frac{1}{\sqrt{\pi v_0}}=4{,}09306 \text{ мм}^{-1};$$

$$n(x=\mu\pm\sigma;v_0)=\frac{1}{\sqrt{\pi v_0}}e^{-1/2}=2{,}48257 \text{ мм}^{-1};$$

$$n(x=\mu\pm 2\sigma;v_0)=\frac{1}{\sqrt{\pi v_0}}e^{-2}=0{,}55394 \text{ мм}^{-1};$$

(24)

$$n(x=\mu\pm 3\sigma;v_0)=\frac{1}{\sqrt{\pi v_0}}e^{-9/2}=0{,}04547 \text{ мм}^{-1}.$$

If we take the density of the value distribution as a unit, then the ratio of the given densities will be as follows:

$$n(x=\mu):n(x=\mu\pm\sigma):n(x=\mu\pm 2\sigma):n(x=\mu\pm 3\sigma)=$$

$$=1:e^{-\frac{1}{2}}:e^{-2}:e^{-\frac{9}{2}}=1:0{,}606:0{,}135:0{,}011.$$

(25)

1.3.About the dispersion of geodetic measurements

A new approach to accuracy assessment was initiated in the INS - 1 recommendation of the International Committee of Weights and Measures, which recommends that the uncertainty components of category A be estimated by sample variances s_i^2 or deviations s_i [6].

It is known that general aggregates of measurements are estimated not by sample variances, which are studied by the theory of probabilities, but by general variances σ^2.

It is necessary to solve the problem of assessing the accuracy of geodetic measurements in the presence of a general set of measurements.

The quantitative (numerical) measure of the deviation of a discrete random variable X from its mathematical expectation $E(X)=\mu$ is the central moment of the second order, which is denoted by the symbol σ^2 (12)

In mathematical statistics, a component of which is the theory of measurement errors, the quantity σ^2 is called the general variance, and the quantity $\sigma = \sqrt{\sigma^2}$ is the standard deviation of the quantity *X*.

The mathematical expectation of the quantity *X* is determined

$$\mu = E(X) = \sum_x x \cdot f(x). \tag{26}$$

In the theory of probabilities, the quantity μ is called the average value of the quantity *X*, and in mathematical statistics – the general average value.

The variance of the discrete quantity *X* is calculated according to the theorem given in the work [5]:

$$\sigma^2 = E(X^2) - \mu^2, \tag{27}$$

were

$$E(X^2) = \sum_x x^2 \cdot f(x) \tag{28}$$

is the mathematical expectation of the square of a random variable, f(x) is a probability distribution function [5].

The proof of (27) is based on definitions (12), (26), (28) and the second axiom of probabilities:

$$\sum_x f(x) = \sum_x p(x) = 1. \tag{29}$$

The second axiom of probabilities is also formulated [2] that $P\{I\}$=1 for a reliable event *I*.

Having given this axiom and pointed out that the equality $P\{E(X)\}$ does not imply that *E* represents a reliable event, probability theory never gives any example of a probable event. Using modern methods that allow you to practically reduce the influence of systematic errors to a minimum, modern devices (high-precision level and invar leveling rails), you can measure the excess between two benchmarks located at a distance of 100 meters from each other by many methods and calculate the general average value μ, which is reliable an interval for that value would not be constructed, that interval would not be likely because its probability is not equal to one.

Probability theory does not pay attention to the range *R* values of measured random variables, because it considers random variables that acquire values in the range from -∞ to ∞. In the theory of measurement errors, the range of measurement values *R* is a probable value, because when the number of measurements of one quantity increases, it approximately acquires a constant value that depends on the accuracy of the measuring device.

Consider the density function of the distribution of the quantity X with mean value μ and variance σ^2 for the normal distribution law (23) [5]:

$$n(x,\mu,\sigma) = \frac{1}{\sqrt{2\pi}\cdot\sigma}\, e^{-\frac{1}{2}\cdot\left[\frac{x-\mu}{\sigma}\right]^2}, \quad -\infty < x < \infty, \qquad (30)$$

де: π = 3.14159265… ; e = 2.71828… .

The theory of probabilities is built on this law, and in geodetic practice random variables acquire the largest values of ±4σ÷±5σ (and sometimes ±6σ).

It follows from definition (12) and theorem (27) that the general variance σ^2 is the dispersion of the quantity X relative to its general mean value μ. The quantity μ is such a numerical implementation of the values of the general set of measurements that has a general variance of σ^2, which is the smallest of all variances that are determined on this set.

For example, if you take any value x_i of the value of X, then it has its own measure of dispersion - dispersion $\sigma^2_{x_i}$.

Definition 3 There is a general set of measurements that have a chi value x_i. The variance $\sigma^2_{x_i}$ of the x_i value of a discrete quantity X is called the mathematical expectation of the square of the deviation of the quantity X from x_i, i.e.

$$\sigma^2_{x_i} = E(X - x_i)^2 = \sum_x (x - x_i)^2 \cdot f(x), \qquad (31)$$

where: $f(x)$ is the probability distribution function [5].

Size $\sigma_{x_i} = \sqrt{\sigma^2_{x_i}}$ is called the standard deviation x_i of the value X values.

Example 3, in table. 1 shows the distribution of the values of the general set of measurements with the volume of n = 100 exceedances between two benchmarks of the leveling course. Let's determine the confidence interval for the values of a discrete quantity, the general average value μ of the excess, the general variance σ^2, the standard deviation $\sigma = \sqrt{\sigma^2}$, as well as the variance: $\sigma^2_{x_{(1)}}, \sigma^2_{x_{(2)}}, \ldots, \sigma^2_{x_{(7)}}$ values $x_{(1)}, x_{(2)}, \ldots, x_{(7)}$

excesses and their standard deviations$\sigma_{x_{(1)}}, \sigma_{x_{(2)}}, \dots, \sigma_{x_{(7)}}$.

The general population has the volume of measurements k = 100, and the volume of "values" of the general population k_G = 7. The degree of quantization [7] of measurements $[Q]$ = xи+1 – xи = 0.1 mm, the range of measurements $R = x_{max} - x_{min}$ = 1 .9 – 1.3 = 0.6 *mm* (Table).

The center of the range of measurements will be as follows:

$$\bar{x}_G = \frac{x_{min}+x_{max}}{2} + \frac{x_{(1)}+x_{(7)}}{2} + \frac{1.3+1.9}{2} = 1.6 \text{ мм}$$

$$\sum_x f(x) = \sum_x p(x) = 0.01 + 0.03 + 0.18 + 0.49 + 0.23 + 0.04 + 0.01 = 1.$$

Taking into account the rounding of the measurement values, we will find the permissible limit values of the excess

$$\begin{cases} x_{adm(min)} = x_{(1)} + \frac{[Q]}{2} = 1.3 - \frac{0.1}{2} = 1.25 \text{ мм} \\ x_{adm(max)} = x_{(7)} + \frac{[Q]}{2} = 1.9 + \frac{0.1}{2} = 1.95 \text{ мм.} \end{cases}$$

The confidence interval for the values of the quantity X will take the form (17):

$$P\left(x_{adm(min)} \leq X \leq x_{adm(max)}\right) = P(1.25 \leq X \leq 1.95) = 1.$$

Using formula (26), we will calculate the general average value of the excess

$$\mu = 1.3 \cdot 0.01 + 1.4 \cdot 0.03 + 1.5 \cdot 0.18 + 1.6 \cdot 0.49 + 1.7 \cdot 0.23 + 1.8 \cdot 0.04 + 1.9 \cdot 0.02 = 1.61,$$

Let's determine the deviation ε of the center of the span $\bar{x}_G$ measurements from the general average value μ

$$\varepsilon = |\bar{x}_G - \mu| = |1.6 - 1.61| = 0.01$$

Since the deviation $\varepsilon<[Q]$ is significantly smaller than the degree of quantization $[Q]$ of the measurements, then in the first approximation it can be assumed that the measurements are normally distributed.

We calculate the general variance of the quantity X according to theorem (27), for this we determine the mathematical expectation of the square of the random variable according to formula (28)

$$E(X^2) = 1.3^2 \cdot 0.01 + 1.4^2 \cdot 0.03 + 1.5^2 \cdot 0.18 + 1.6^2 \cdot 0.49 + 1.7^2 \cdot 0.23 + 1.8^2 \cdot 0.04 + 1.9^2 \cdot 0.02 = 2.6016 \text{ мм}^2,$$

so,

$$\sigma^2 = 2.6016 - 1.61^2 = 0.0095 \text{ мм}^2 \;\; \sigma = \sqrt{0.0095} = 0.0975 \text{ мм}.$$

Let's calculate the scattering using definition (31): $\sigma^2_{x_{(1)}}, \sigma^2_{x_{(2)}}, \ldots, \sigma^2_{x_{(7)}}$ values $x_{(1)}$, $x_{(2)}$, …, $x_{(7)}$ general set of measurements and their standard deviations$\sigma_{x_{(1)}}, \sigma_{x_{(2)}}, \ldots, \sigma_{x_{(7)}}$,

get possessed

$\sigma^2_{x_1} = \sum_x (x - x_1)^2 = (x_2 - x_1)^2 \cdot f(x_2) + (x_3 - x_1)^2 \cdot f(x_3) + (x_4 - x_1)^2 \cdot f(x_4) + (x_5 - x_1)^2 \cdot f(x_5) + (x_6 - x_1)^2 \cdot f(x_6) + (x_7 - x_1)^2 \cdot f(x_7) = (1.4 - 1.3)^2 \cdot 0.03 + (1.5 - 1.3)^2 \cdot 0.18 + (1.6 - 1.3)^2 \cdot 0.49 + (1.7 - 1.3)^2 \cdot 0.23 + (1.8 - 1.3)^2 \cdot 0.04 + (1.9 - 1.3)^2 \cdot 0.02 = 0.1056 \text{ мм}^2$;

$\sigma^2_{x_2} = \sum_x (x - x_1)^2 = (x_1 - x_2)^2 \cdot f(x_1) + (x_3 - x_2)^2 \cdot f(x_3) + (x_4 - x_2)^2 \cdot f(x_4) + (x_5 - x_2)^2 \cdot f(x_5) + (x_6 - x_2)^2 \cdot f(x_6) + (x_7 - x_2)^2 \cdot f(x_7) = (1.3 - 1.4)^2 \cdot 0.01 + (1.5 - 1.4)^2 \cdot 0.18 + (1.6 - 1.4)^2 \cdot 0.49 + (1.7 - 1.4)^2 \cdot 0.23 + (1.8 - 1.4)^2 \cdot 0.04 + (1.9 - 1.4)^2 \cdot 0.02 = 0.0536 \text{ мм}^2$.

Similarly, we will obtain $\sigma^2_{x_3} = (x \quad - x_3)^2 \cdot f(x \quad) = 0.0216 \text{ мм}^2$;

$\sigma^2_{x_4} = (x - x_4)^2 \cdot f(x) = 0.0096 \text{ мм}^2$; $\sigma^2_{x_5} = (x - x_5)^2 \cdot f(x) = 0.0176 \text{ мм}^2$;

$\sigma^2_{x_6} = (x - x_5)^2 \cdot f(x) = 0.0456 \text{ мм}^2$; $\sigma^2_{x_7} = (x - x_7)^2 \cdot f(x) = 0.0936 \text{ мм}^2$.

The calculation of the dispersions $\sigma^2_{x_i}$ of the values of the quantity X will be significantly simplified if the general rule for their calculation is applied.

Theorem 1. If μ is the general average value, and σ^2 is the general variance of the discrete quantity X, then the variance $\sigma^2_{x_i}$. of the value of x_i will take the form:

$$\sigma^2_{x_i} = \sigma^2 + (x_i - \mu)^2. \tag{32}$$

Proof. By definition 3 (31)

$$\sigma^2_{x_i} = \sum_x (x - x_i)^2 \cdot f(x) = \sum_x \left(x^2 - 2xx_i + x_i^2\right) f(x) =$$
$$= \sum_x x^2 f(x) - 2x_i \sum_x x f(x) + \sum_x x_i^2 f(x). \tag{33}$$

In this equation $\sum_x x_i^2 f(x) = x_i^2 \sum_x f(x)$, taking into account the axiom of probabilities (29), we obtain: $\sum_x x_i^2 f(x) = x_i^2$, based on definitions (26), (28) in the equation (33):

$\sum_x x f(x) = \mu$, a $\sum_x x^2 f(x) = E(X^2)$, thus, equation (33) is simplified:

$$\sigma_{x_i}^2 = E(X^2) - 2x_i\mu + x_i^2 \qquad (34)$$

Taking into account the theorem (27): $E(X^2) = \sigma^2 + \mu^2$, using this dependence, formula (34) can be written in the following form:

$$\sigma_{x_i}^2 = \sigma^2 + \left(x_i^2 - 2x_i\mu + \mu^2\right) = \sigma^2 + (x_i - \mu)^2, \qquad (35)$$

Thus, the theorem is proved.

Two consequences follow from the theorem.

Corollary 1. If the general set of measurements has a degree of quantization $[Q]$, then the variance $\sigma_{x_i}^2$. of the value x_i of the quantity X will have the form:

$$\sigma_{x_{(i)}}^2 = \sigma^2 + \left[\left(x_{(1)} + (i-1)[Q]\right) - \mu\right]^2, \qquad (36)$$

were : $x_{(1)} = x_{\min}$ is the first of a series of «values» of a set of dimensions ordered by growth; and i -is the serial number of the «value» of the measurement in the series of measurements.

If there is a general set of measurements, then the series of «values» of the measurements ordered in ascending order has the degree of quantization $[Q]$, therefore,

$$x_{(2)} = x_{(1)} + [Q];\ \ x_3 = \ 2[Q]; \dots;$$

$$x_{(k_{(G-1)})} = x_{(1)} + (k_G - 2)[Q];\ \ x_{(k_{(G)})} = x_{(1)} + (k_G - 1)[Q],$$

in this series, the value $(k_G\text{ -}1)[Q]=R$. Taking into account the given values, we proceed from rule (32) to rule (36), so the dispersion of measurements depends on their range R and the degree of quantization $[Q]$.

Corollary 2. If there is a random sample of the volume k, then the sample variance $s_{x_i}^2$. of the value x_i of the quantity X will have the form:

$$s_{x_i}^2 = s^2 + (x_i - \bar{x})^2, \qquad (37)$$

where: $s^2 = \frac{1}{k}\sum_{i=1}^{k}(x_i - \bar{x})^2$, which is the sample variance [2] ; $\bar{x} = \frac{1}{k}\sum_{i=1}^{k} x_i$ – it sample mean value (arithmetic mean) of the value *X*.

The sample variance s^2 is an estimate for the general variance σ^2, and the algebraic representation of the mean value μ of the quantity *X* and the variance $\sigma_{x_i}^2$ in a random sample will be, respectively, the sample mean value $\bar{x}$ and the sample variance $s_{x_i}^2$, taking into account the following correspondence from rule (32), we arrive at rule (37).

Example 2. Let's take the distribution series from example 1 with its characteristics. It is necessary to calculate the dispersion of measurement values according to the given theorem (32), rule (36) and standard deviations of measurement values, so we have:

$$\sigma_{x_1}^2 = \sigma^2 + (x_1 - \mu)^2 = 0.0095 + (1.3 - 1.61)^2 = 0.1056 \text{ мм}^2,$$

$$\sigma_{x_2}^2 = \sigma^2 + (x_2 - \mu)^2 = 0.0095 + (1.4 - 1.61)^2 = 0.0536 \text{ мм}^2,$$

we find similarly:

$\sigma_{x_3}^2 = \sigma^2 + (x_3 - \mu)^2 = 0.0216 \text{ мм}^2$; $\sigma_{x_4}^2 = \sigma^2 + (x_4 - \mu)^2 = 0.0096 \text{ мм}^2$;

$\sigma_{x_5}^2 = \sigma^2 + (x_5 - \mu)^2 = 0.0176 \text{ мм}^2$; $\sigma_{x_6}^2 = \sigma^2 + (x_6 - \mu)^2 = 0.0456 \text{ мм}^2$;

$$\sigma_{x_7}^2 = \sigma^2 + (x_7 - \mu)^2 = 0.0936 \text{ мм}^2.$$

Let's calculate the variance of the values of the general population according to the rule (36):

$$\sigma_{x_{(1)}}^2 = 0.0095 + [(1.3 + (1 - 1)(0.1) - 1.61)]^2 = 0.1056 \text{ мм}^2;$$

$$\sigma_{x_{(2)}}^2 = 0.0536 \text{ мм}^2;\ \sigma_{x_{(3)}}^2 = 0.0216 \text{ мм}^2;\ \sigma_{x_{(4)}}^2 = 0.0096 \text{ мм}^2;$$

$$\sigma_{x_{(5)}}^2 = 0.0176 \text{ мм}^2;\ \sigma_{x_{(6)}}^2 = 0.0456 \text{ мм}^2;\ \sigma_{x_{(7)}}^2 = 0.0936 \text{ мм}^2.$$

Therefore, if the variances $\sigma_{x_i}^2$. of the values of the quantity *X* are determined by features (31), by theorem (32) or by rule (36), then it can be seen that they have the same values.

The deviation of the measurement values will be:

$$\sigma_{x_{(1)}} = 0.325 \text{ мм};\ \sigma_{x_{(2)}} = 0.232 \text{ мм};\ \sigma_{x_{(3)}} = 0.147 \text{ мм};\ \sigma_{x_{(4)}} = 0.098 \text{ мм};$$

$$\sigma_{x_{(5)}} = 0.133 \text{ мм};\ \sigma_{x_{(6)}} = 0.214 \text{ мм};\ \sigma_{x_{(7)}} = 0.306 \text{ мм};$$

We will control the calculation of the dispersion values of the quantity *X* according to the following theorem.

Theorem 2. If the general set of measurements has the volume *k*, the measurement of the quantity *X* is the volume k_G, the general variance σ^2 and the sum of the variances $\sigma_{x_i}^2$. of the values of the measurements, the variances $\sigma_{x(i)}^2$. of the values of the quantity *X* have the following dependencies:

$$\sigma^2 = \frac{\sum_{i=1}^{k} \sigma_{x_i}^2}{2k} \qquad (i = 1, 2, \dots, k); \qquad (38)$$

$$\sigma^2 = \frac{\sum_{i=1}^{k_G} \sigma_{x(i)}^2 f(x_i)}{2} \qquad (i = 1, 2, \dots, k_G), \qquad (39)$$

where: $f(x_{(i)})$ are the probabilities of the value $x_{(i)}$ of the quantity *X*.

It follows from Theorem 1 of equation (18) that

$$\sigma_{x_1}^2 = \sigma^2 + x_1^2 - 2x_1\mu + \mu^2;$$

$$\sigma_{x_2}^2 = \sigma^2 + x_2^2 - 2x_2\mu + \mu^2;$$

. . . .

$$\sigma_{x_k}^2 = \sigma^2 + x_k^2 - 2x_k\mu + \mu^2,$$

We add the left and right parts of these equations, we get

$$\sum_{i=1}^{k} \sigma_{x_i}^2 = k\sigma^2 + \sum_{i=1}^{k} \sigma_{x_i}^2 - 2\sum_{i=1}^{k} x_i\mu + k\mu^2,$$

let's divide the left and right parts of this equation by *k*, we get

$$\frac{\sum_{i=1}^{k} \sigma_{x_i}^2}{k} = \sigma^2 + \frac{\sum_{i=1}^{k} x_i^2}{k} - 2\frac{\sum_{i=1}^{k} x_i}{k}\mu + \mu^2 =$$

$$= \sigma^2 + E(X^2) - 2E(X)\mu + \mu^2 = \sigma^2 + E(X^2) - \mu^2,$$

taking into account theorem (27), we obtain $\frac{\sum_{i=1}^{k} \sigma_{x_i}^2}{k} = 2\sigma^2$, from this equation we arrive

to dependence (38).

Let's assume that the general population matters $x_{(1)}$, $x_{(2)}$, …, $x_{(k_G)}$, Sumy

variances of measurements that have the same values $\sigma_{x_{(1)}}^2, \sigma_{x_{(2)}}^2, \dots, \sigma_{x_{(k_G)}}^2$, will be

such:

$$\sigma^2_{x_{(1)}} kf\left(x_{(1)}\right);\ \sigma^2_{x_{(1)}} kf\left(x_{(1)}\right);\ \dots,\ \sigma^2_{x_{(k_G)}} kf\left(x_{(k_G)}\right).$$

So, $\cdot \sum_{i=1}^{k_G} \sigma^2_{x_{(i)}} f\left(x_{(i)}\right)$, will be the sum of the variances of the values of the general aggregates, taking into account the dependence (38), we obtain

$$\sigma^2 = \frac{k \sum_{i=1}^{k_G} \sigma^2_{x_{(i)}} f\left(x_{(i)}\right)}{2k} = \frac{\sum_{i=1}^{k_G} \sigma^2_{x_{(i)}} f\left(x_{(i)}\right)}{2},$$

Accordingly, Theorem 2 is proved. Two consequences follow from the theorem.

Corollary 1. If there is a general set of measurements, then the average variance ν_0^2 of the values of a discrete quantity and its average standard deviation ν_0 have the following form:

$$\nu_0^2 = 2\sigma^2; \tag{40}$$

$$\nu_0 = \sigma\sqrt{2}. \tag{41}$$

From equations (38) and (39), we obtain:

$$2\sigma^2 = \frac{\sum_{i=1}^{k} \sigma^2_{x_i}}{k} \qquad (i = 1, 2, \dots, k\ ); \tag{42}$$

$$2\sigma^2 = \sum_{i=1}^{k_G} \sigma^2_{x(i)} f(x_i) \quad (i = 1, 2, \dots, k_G). \tag{43}$$

From equations (42) and (43) it follows that the quantity $2\sigma^2$ is the average variance of the values of the quantity *X*. Denoting the average deviation of the quantity *X* with the symbol ν_0, from equations (42) and (43) we obtain equations (40) and (41).

Corollary 2. If there is a random sample that represents the general set of measurements, then the average sample variance ν^2 and the average sample deviation ν of the quantity *X* will take the following form:

$$\nu^2 = 2s^2; \tag{44}$$

$$\nu = s\sqrt{2}. \tag{45}$$

The sample variance s^2 is a direct algebraic reflection of the general variance σ^2. So, from dependencies (40) and (41) we arrive at correspondences (44) and (45).

The general dispersion σ^2 is considered to be a characteristic of the dispersion of the quantity *X*. In fact, the quantity *X* is the square of the mean squared deviation of the values of this quantity from its mean value μ. The average variance ν_0^2ν is a numerical realization of the set of variances of all values of the quantity *X*, therefore the average variance ν_0^2 is a better estimate of the dispersion of the quantity X than the general variance σ^2.

Example 3. According to the series of distribution of the values of the general set of measurements of the excess given in the table. 1, it is necessary to control the calculations, determine the average variance and average deviation of the quantity X, for this we will find the volumes m_1, m_2, . . ., m_7 dimensions that matter $x_{(1)}$, $x_{(2)}$,. . ., $x_{(7)}$:

$$m_1 = kf(x_1) = 100 \cdot 0.01 = 1;\ m_2 = kf(x_2) = 100 \cdot 0.03 = 3;$$

$$m_3 = kf(x_3) = 100 \cdot 0.18 = 18;\ m_4 = kf(x_4) = 100 \cdot 0.49 = 49;$$

$$m_5 = kf(x_5) = 100 \cdot 0.23 = 23;\ m_6 = kf(x_6) = 100 \cdot 0.04 = 4;$$

$$m_7 = kf(x_7) = 100 \cdot 0.02 = 2;$$

CONTROL: $\sum_{i=1}^{7} = 1 + 3 + 18 + 49 + 23 + 4 + 2 = 100$, Let's find the sum of the dispersion values of the quantity X

$$\sum_{i=1}^{7} \sigma_{x_i}^2 = m_1\sigma_{x_1}^2 + m_2\sigma_{x_2}^2 + \cdots + m_7\sigma_{x_7}^2 = 1.9 \text{ мм}^2.$$

Control of the calculation of the variance of the values of the quantity X will be carried out according to the formulas (38) and (39):

$$\sigma^2 = \frac{\sum_{i=1}^{k} \sigma_{x_i}^2}{2k} = \frac{1.9}{2 \cdot 100} = 0.0095 \text{ мм}^2$$

$$\sum_{i=1}^{7} \sigma_{x_i}^2 = 0.1056 \cdot 0.01 + 0.0536 \cdot 0.03 + 0.0216 \cdot 0.18 + 0.0096 \cdot 0.49 +$$

$$+0.0176 \cdot 0.23 + 0.0456 \cdot 0.04 + 0.0936 \cdot 0.03 = 0.019 \text{ мм}^2$$

$$\sigma^2 = \frac{\sum_{i=1}^{k_G} \sigma_{x(i)}^2 f(x_i)}{2} = \frac{0.019}{2} = 0.095 \text{ мм}^2 \qquad (i = 1, 2, \dots, k_G).$$

Using formulas (40) and (41), we find the average variance and the average deviation of the value X:

$$\nu_0^2 = 2\sigma^2 = 2 \cdot 0.095 = 0.019 \text{ мм}^2;\ \nu_0 = 0.0975 \cdot \sqrt{2} = 0.138 \text{ мм}.$$

Summing up, we can say that the obtained formulas (32) and (36) can be used to quickly determine the variance of the values of the general sets of measurements and establish the weights of these measurements in relation to the closedness of geodesic constructions [4], this makes it possible to distribute inviscid directly into the values of chi measurements and to obtain more accurate general average values of measurements.

The resulting formulas (40), (41), (42), (43) make it possible to find the average variances and average deviations of measurements of general and sample sets of measurements.

Conclusions. Properties of true errors.

1.In a closed equalized system, true errors do not accumulate, but are compensated.

Conclusions:

2. Two statements of the theory of measurement errors are substantiated.

3. The average variance is a general characteristic of the dispersion of a random variable. It is a better estimate of the dispersion of a value than the general variance

4. Correspondences (23) are properties of the normal distribution curve.

Prospects for further investigations in this direction are to establish distribution laws, properties of position characteristics and dispersion of random variables that acquire values in the point set of the interval (17).

Conclusion generel. The dispersion values of geodetic measurements, and therefore the accuracy of the measurements, depend on the degree of quantization $[Q]$ and the range R of the measurements. High-precision measurements are usually performed with a constant degree of quantization. Therefore, in order to reduce the dispersion of measurements, measurements must be carried out with such devices, the use of which reduces the range of R measurements, for example, in the case of angular measurements and leveling, smaller ranges of measurements are obtained when using total stations, theodolites and levels, the sight tubes of which have larger diameters of the inlet holes.

Prospects for further research in this direction are in the development of new methods for equalizing measurements of elements of closed systems of circles K, M of geodesic constructions [7], in determining the accuracy of equalized elements of such systems, in finding the most probable values of dispersions of the results of adding aggregates of measurements and in the established laws of distribution [8, 9] discrete values that take on values within their probable range R.